

# Positional averaging explains crowding with letter-like stimuli

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Visual crowding is a breakdown in object identification that occurs in cluttered scenes, a process that represents the principle restriction on visual performance in the periphery. When crowded objects are presented experimentally, a key finding is that observers frequently report nearby flanking items instead of the target. This observation has led to the proposal that crowding reflects increased noise in the positional code for objects; although how the presence of nearby objects might disrupt positional encoding remains unclear. We quantified this disruption using cross-like stimuli, where observers judged whether the horizontal target line was positioned above or below the stimulus midpoint. Overall, observers were poorer at judging position in the presence of crowding flankers. However, offsetting horizontal lines in the flankers also led observers to report that the horizontal line in the target was shifted in the same direction, an effect that held for subthreshold flanker offsets. In short, crowding induced both random and systematic errors in observers' judgment of position, with or without the detection of flanker structure. Computational modeling reveals that perceived position in the presence of flankers follows a weighted average of noisy target- and flanker-line positions, rather than a substitution of flanker-features into the target, as has been proposed previously. Together, our results suggest that crowding is a preattentive process that uses averaging to regularize the noisy representation of position in the periphery.

peripheral visual field | texture recognition | context | lateral interaction

When multiple objects are presented in close proximity, their identities can become obscured through a process known as crowding (1, 2). This impairment in identification occurs across a wide range of visual modalities (3–6) and increases in magnitude when target items are presented peripherally, making crowding the principle restriction on our representation of the peripheral visual field (7–9). Although a range of theories have been proposed to explain crowding (reviewed recently in ref. 8), there is little consensus regarding their plausibility, and a scarcity of computational models that can account for the effects of crowding with complex stimuli such as letters.

One of the most striking features of crowding is that observers' identification errors frequently arise from them reporting the identity of a flanking item instead of the target (10–14). Such errors are particularly apparent with multifeature stimuli such as letters (15), where the subjective experience of crowding is that features from each object become “jumbled” (9). This observation has led to the proposal that crowding results from noise in the encoding of position, leading to an intermixing of either the feature positions within each object (12, 16), or the gross position of objects as a whole (14, 17). These proposals are supported by the correlation between the magnitude of crowding—which increases as objects move further into the peripheral visual field (1)—and the decline in position encoding in the periphery (18–20). Similarly, errors for both absolute judgments of position (21) and saccades to eccentric targets (22) exhibit the same asymmetric pattern as crowding (23, 24), with greater errors

along the radial axis from fixation than along the tangential (i.e., orthogonal) axis. It follows that the intermixing of target and flanker features under crowding could simply be the result of the positional uncertainty inherent to peripheral vision.

However, despite the apparent relationship between positional uncertainty and crowding, the precise effect of crowding on the perceived position of objects and their features remains unclear. Although crowding has been shown to affect judgments of relative position in vernier tasks, with thresholds rising sharply in the presence of additional flanking lines (25, 26), vernier acuity also relies upon luminance contrast and orientation information (27, 28), making it unclear whether poor performance reflects positional uncertainty per se. The range and selectivity of these interactions also differs from patterns established with crowding in other domains (9), suggesting that the effects on vernier acuity may arise from processes more akin to masking than crowding. The aim of the present study was thus to directly examine the influence of crowding on the perceived position of object features.

Of the many theories proposed to explain crowding, 2 clear predictions emerge for the effect of crowding on position encoding. First, the disproportionate amount of flanker-related errors (10–14) suggests that crowding arises from the features of flankers (or flankers in their entirety) being substituted into the target. This substitution has been attributed to both low-level processes such as positional uncertainty (12, 13, 16, 17), and higher-order operations such as unfocused spatial attention (10), a distinction that follows a more general debate regarding the locus of crowding (8). A second prediction emerges from the view that crowding is a preattentive and compulsory averaging of target and flanker features. For example, crowded Gabor elements can contribute to a global estimate of orientation (29), although the exact combination of elements may follow more directed pooling schemes according to the modality involved (5, 30). Were this model to apply in the positional domain, the position of target and flanking objects would be averaged, an effect that may serve to simplify the appearance of noisy positional structure in the periphery.

To examine the influence of crowding on position encoding and so distinguish between “averaging” and “substitution” hypotheses, observers were presented with cross-like stimuli, and judged whether the horizontal target line was above or below the stimulus midpoint (see *Methods* and Fig. 1A). Stimuli were “letter-like” in that they were multifeatured and had proportions equivalent to those of Sloan letters. An additional experiment required observers to make a continuous (i.e., nondiscrete) estimate of the position of target features, allowing a more

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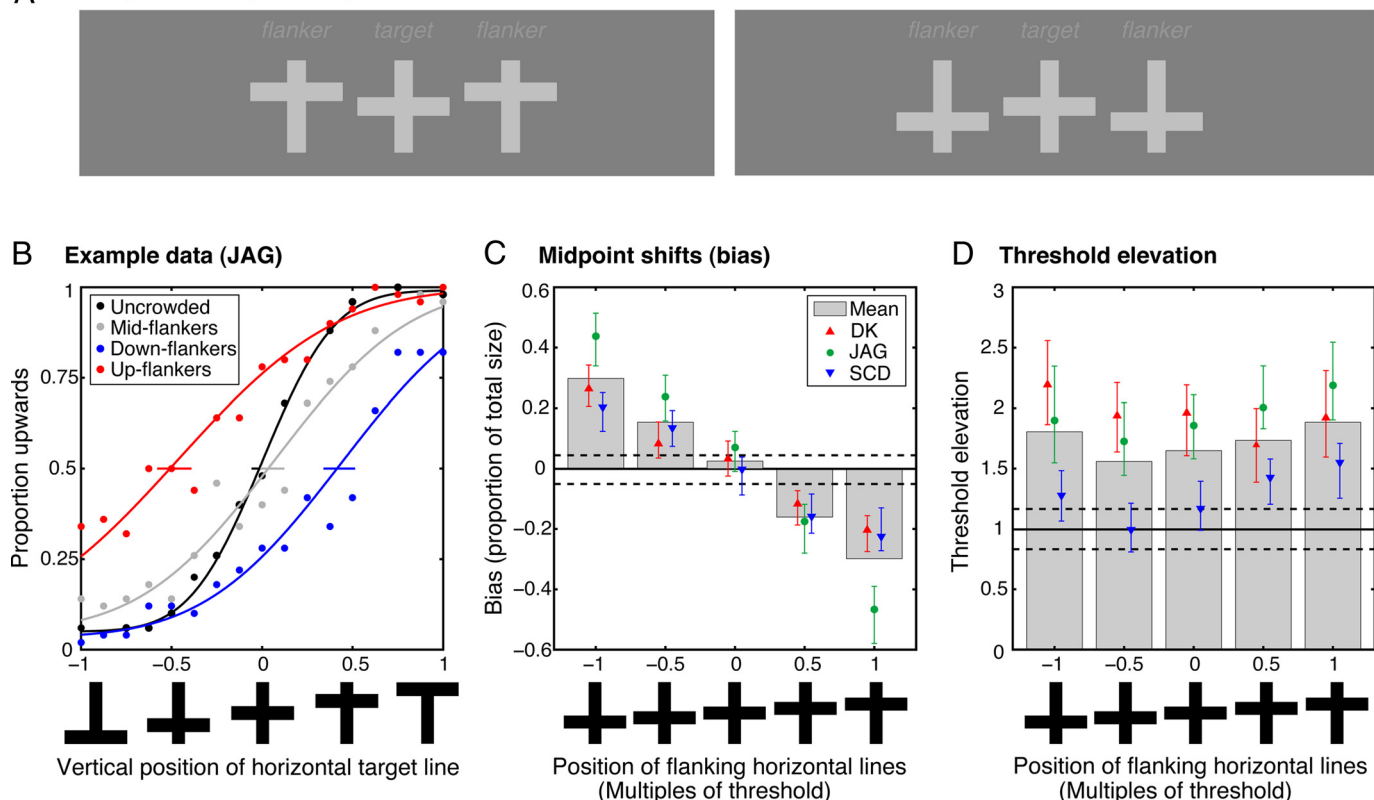
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### A Example stimuli (two sets)



**Fig. 1.** The effects of crowding on position encoding. (A) Example stimuli, with the middle target-cross depicted with the horizontal line centered along the vertical axis. Flankers on the left have horizontal lines positioned above the midpoint (up-flankers), with the converse down-flankers on the right. The effect of crowding can be seen by fixating near the bottom of the figure caption: Target-horizontals should appear shifted toward the flanker-horizontals. (B) Data from 1 observer, with the proportion of 'upward' responses plotted as a function of the vertical position of the target-horizontal (shown schematically on the abscissa), and fit with psychometric functions. 95% confidence intervals are depicted around the midpoints, obtained from bootstrapping. Data are presented for targets in isolation (uncrowded), and crowded conditions with midflankers and up- and down-flankers offset by threshold-level displacements. Crowding both increases the slope and shifts psychometric functions to a degree that depends on flanking structure. (C) Midpoints of the psychometric functions (bias) for 3 observers in each condition, expressed as a proportion of stimulus size (scaled to  $\pm 1$ ) and plotted as a function of the vertical position of flanker-horizontals (expressed in multiples of the uncrowded threshold for position discrimination). Mean data are presented as gray bars, with individual points superimposed; error bars (and hatched lines for uncrowded data) depict 95% confidence intervals. Positive shifts in flanker feature positions (even subthreshold ones) produce negative shifts in bias, indicating an increase in 'upward' responses with up-flankers, and vice versa. (D) As in 1C for threshold elevation. The presence of flankers produced a strong elevation of positional thresholds, regardless of their structure.

detailed comparison of the predictions of averaging and substitution models to human performance.

## Results

In the absence of flankers, observers' judgments of the position of a horizontal target line (above or below the midpoint of a cross-like element; Fig. 1A) were unbiased, that is, their reports were symmetrically distributed around the stimulus midpoint (Fig. 1B, black curve). When 2 flankers were present, each with horizontal lines positioned at the midpoint (midflankers), psychometric functions become shallower (Fig. 1B, pale-gray curve), indicating that larger shifts were needed to report the position of the target-horizontal reliably (i.e., position discrimination thresholds were elevated). The midpoint of functions remained unchanged (i.e., centered target-horizontals still appeared centered when crowded). Shallow functions are also evident in conditions where flanking features were displaced above or below the midpoint (up- and down-flankers, red and blue curves in Fig. 1B, respectively) by a threshold-level displacement (i.e., a positional shift that was just noticeable for uncrowded elements). Furthermore, up-flankers also shift the entire curve leftward (indicating more 'up' responses), which results in a negative midpoint value as the subjective midpoint

(where observers were equally likely to say 'up' or 'down') is now located at a negative (downward) position on the cross. The opposite pattern occurred with down-flankers, where an increased probability of 'down' responses results in a rightward shift of the psychometric function.

These midpoint/bias values are plotted in Fig. 1C, highlighting the finding that flankers with both subthreshold- and threshold-level displacements below the stimulus midpoint produced an increase in ‘downward’ responses, and a corresponding positive shift in bias. This result demonstrates that judgments were increasingly made in a direction consistent with flanking structure, which is also true of the up-flanker conditions, where negative bias is evident for both displacements. Additionally, the bias induced by subthreshold feature displacements ( $0.5\times$  threshold) demonstrates that crowding can alter position encoding even when the flankers are difficult to distinguish from those that do not produce bias (i.e., midflankers). That is, although offsets in the subthreshold flanking-horizontals could be correctly identified as offset from the midpoint at only 13.3% above chance (based on uncrowded performance, making the actual percentage lower because of the crowding of flankers by the target; 31), they nonetheless produced a significant level of bias in observers’ responses to the crowded target. Importantly, the

more commonly observed effects of crowding are also apparent in our data, with all configurations producing an equivalent level of threshold elevation (Fig. 1D). These effects were also apparent when observers judged the position of the vertical line (left/right), albeit with less induced bias and higher threshold elevation (supporting information and Fig. S1), which rules out the possibility that our results were caused by observers inappropriately using the flanker-horizontals as a coaligned reference. The effect of crowding on midpoints and thresholds also declined with increasing target-flanker separation according to the ‘Bouma law’ of spatial extent (1, 9), with interference zones of 4–6° (0.3–0.4× eccentricity; Fig. S2). This similarity in spatial extent is consistent with both effects arising from a common source related to crowding. In sum, our data show that the crowding of positional information involves both an elevation in thresholds for position discrimination and a systematic bias that shifts the position of target elements to resemble flanking features.

We next sought to determine the precise mechanism underlying these effects. As detailed above, 2 predictions can be made for the encoding of position under crowded conditions. First, substitution processes would result in either the features of flanking items, or flankers in their entirety, being substituted into the target (10, 12, 16) so that the perceived position of target features should alternate between the veridical location and that of the flanking elements, producing a bimodal distribution of perceived feature positions. Second, were averaging (29) to apply in the positional domain, the perceived position of target features should lie at the average of target and flanker feature positions. Although more complex averaging schemes could alter the precise weights assigned to target and flanker positions (5, 30), the perceived target positions should nonetheless show a unimodal distribution centered on an intermediate value. As both models could produce the bias in Fig. 1 (Fig. S3), a more precise examination of *perceived* feature positions is required to differentiate between these accounts.

In Experiment 2, the horizontal target line was again presented at a range of positions, although flanker-horizontals were now positioned *relative to the target* (when crowded), either above or below the target line at either 2 or 4 times the threshold for positional discrimination. This procedure allowed a continuous range of both target and flanker positions, reducing the likelihood that observers would preferentially indicate particular feature positions (as they might were specific positions more likely to be present). Following each trial, a response cross-element appeared near fixation and observers adjusted the horizontal-line position until it matched their percept of the target. Results are presented in Fig. 2A, pooled across observers. With single targets (*Left*), the perceived position of the horizontal feature was largely veridical (lying along the white line), although responses are clearly repelled away from the stimulus midpoint to more extreme positions (i.e., observers rarely reported the target as a ‘+’). This *midpoint repulsion* appears similar to subjects’ general reluctance to report the reference in discrimination tasks (31, 32). In our experiment, the stimulus centroid (33, 34) is likely to have served as the reference. Importantly, because this repulsion occurs with uncrowded elements, it cannot be explained by decision processes such as the ‘signed-max’ rule that depend on stimulus set size (35), nor can it explain the effects of crowding. In the presence of flankers (*Center* and *Right* of Fig. 2A), responses were pulled toward that of the flanking features. Because the results were identical for both up-flankers and down-flankers, the latter results were inverted and pooled with those of the up-flankers to compute the data in Fig. 2A. With a small offset in flanking features (*Center*), responses fell both on and between the flanking and target positions, whereas with large offsets (*Right*) these responses clearly lie between the target and flanker positions. The latter is

consistent with observers relying on the *average* of the target and flanker feature positions, as opposed to the probabilistic substitution of flanker features. However, because of the effects of positional uncertainty and midpoint repulsion, we also sought to test this proposition formally using simulation.

The predictions of averaging and substitution models of crowding were compared using a 3-stage model of each process (depicted in Fig. 2B, see *Methods*). Both began by registering feature positions with added positional noise, reflecting raised positional uncertainty in the periphery (18–20). This stage was unaffected by crowding, consistent with the invariance of contrast-detection thresholds for crowded stimuli (9, 36). The second stage involved the computation of either a weighted average of target and flanker positions or a probabilistic substitution of flanker features into the target. Both the strength of weighting and the probability of substitution were free to vary. Finally, crowded target-position estimates were pushed away from the midpoint to simulate the observed midpoint repulsion. For each model, the best-fitting parameters were selected as those that minimized the least-squares error between the predicted responses and those plotted in Fig. 2A.

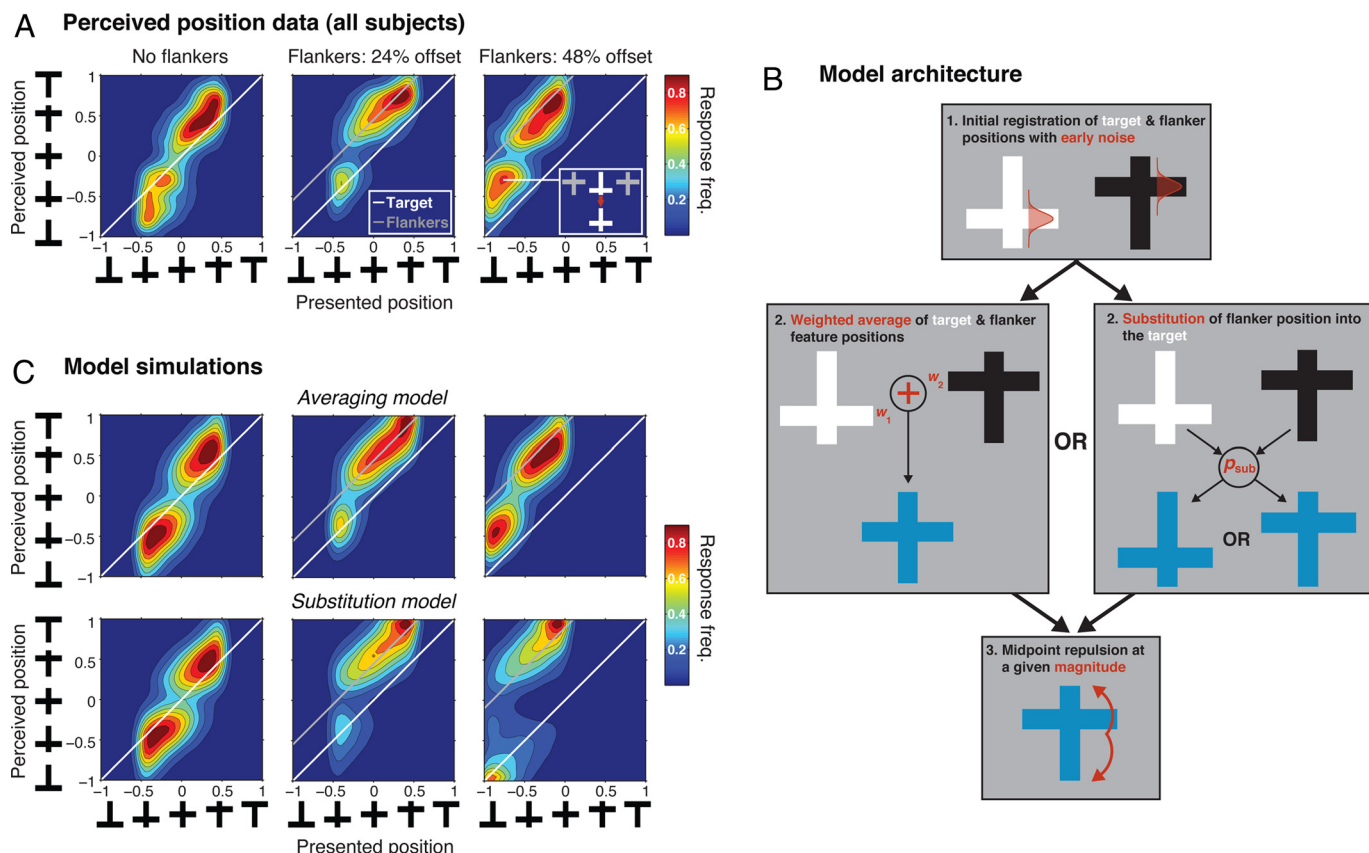
These simulated responses for both averaging and substitution are plotted in Fig. 2C. In the absence of flankers (*Left*), both models correctly predict a percept of the targets that is close to veridical, albeit subject to positional uncertainty and repulsion away from the stimulus midpoint. Under crowded conditions, the averaging model produces estimates that initially lie on a line running parallel between the target and flanker positions (see Fig. S4), with the final-stage repulsion then pushing these responses away from the midpoint. With a small feature offset (*Center*), the final predicted responses lie along the target and flanker lines, because of the repulsion of the averaged values. Larger offsets (*Right*) produce responses that are intermediate between the target and flanking positions, closely replicating the pattern in our data. By contrast, predictions from the substitution model were initially distributed bimodally between the target and flanker positions, before the midpoint repulsion in the final stage. Although this process produces a pattern that approximates our data with small flanker offsets (*Center*), the predicted values for large flanker offsets (*Right*) fall at considerably more extreme positions than those perceived by our observers. Substitution is thus inconsistent with the data in this condition, in comparison with the close match produced by a weighted average of the noisy target- and flanker-feature positions.

It is important to note that neither of these models adds any noise in the second (crowding) stage, each adding only bias from the flanking features. Indeed, the addition of a second noise parameter to these simulations did not produce a significant improvement in the predicted responses (see *Methods*). At first glance, this lack of additional noise would appear to conflict with our finding that crowding induces both bias and an elevation of thresholds (Fig. 1). However, model simulations of Experiment 1 demonstrate that threshold elevation does arise from the combination of a noisy representation of the target position with an equally noisy representation of the flanker feature positions (see Fig. S3). An averaging of target and flanker positions can thus account for both the bias and the threshold elevation observed in the perceived position of crowded target features.

## Discussion

We have demonstrated that crowding has 2 main effects on the perceived position of object features. First, crowding elevates thresholds for the discrimination of feature positions, consistent with the performance impairments seen for vernier acuity (25, 26) and across a range of other modalities (3–6). Second, crowding produces a systematic shift in the perceived position of object features, bringing them into alignment with those of





flanking objects. When the perceived position of these crowded features was examined, in conjunction with computational modeling of these processes, the observed responses were predicted by a weighted average of position estimates, as opposed to the substitution processes more commonly discussed in the positional domain. This finding is broadly consistent with the averaging observed when oriented Gabor elements are crowded (29), although the weights incorporated into our averaging model can also account for the modulatory effects of target-flanker similarity (4, 9) and target-flanker separation (23). The necessary inclusion of midpoint repulsion in our model is also consistent with prior research suggesting that a simple averaging approach is unlikely to completely encapsulate crowding in other domains (5, 30).

Although substitution processes are typically invoked to explain phenomena such as the high proportion of flanker-related errors in crowding tasks (10–14), an averaging of feature positions between target and flanker objects would also give rise to these effects. This outcome is particularly likely given the nonlinear weighting in our model that can produce effects akin to substitution with an extreme weighting toward the flankers. These simulations also demonstrate the difficulty in separating

predictions from averaging and substitution models, with the combination of midpoint repulsion and high positional uncertainty producing identical predictions in the presence of flankers that differ from targets by small positional offsets. Both models made identical predictions for the results of Experiment 1, demonstrating that both a fine response scale and large positional offsets are required to disambiguate averaging and substitution accounts of crowding. This procedural issue could explain the lack of averaging observed with oriented Gabor stimuli using a coarse response scale (31), which would make the presence of near-averaged stimuli difficult to determine. It is only with large positional offsets and a fine response scale that the predictions from the substitution model can be shown to be untenable.

Importantly, neither of the models examined added any noise to the operations associated with crowding, each adding only bias from the flanking features on any given trial. Both the systematic bias and random effects associated with crowding could thus be attributed to the combination of a noisy representation of the target feature position with noisily represented flanker feature positions. In other words, the deleterious effects of crowding on position encoding can be accounted for by simple recourse to the

inherent positional uncertainty in the peripheral visual field (18–20). This aspect of our model is similar to a recent model of feature binding (37), where the high positional noise in the periphery causes target features to be erroneously associated with adjacent elements within coarse feature maps. Additionally, because there are also intrinsic levels of uncertainty in other domains affected by crowding (3–6), such as orientation (38) and direction (39), our model raises the intriguing possibility that crowding could involve the averaging of features across all affected modalities, as opposed to the addition of noise suggested by the historic focus on error rates in crowding tasks.

As a whole, our results are most consistent with a preattentive basis for crowding. The observation that crowding can occur with flanker offsets below the threshold for positional discrimination (Fig. 1) in particular suggests that crowding does not require an awareness of flanking structure. Although we cannot rule out the possibility that this bias arose from the veridical flanker displacements reaching awareness on a small proportion of trials (e.g., 40), the nature of this bias is also inconsistent with the mechanisms typically proposed to link attention with crowding. Attentional theories proposing that crowding causes the loss of target identities from awareness (41) would predict random responses (i.e., threshold elevation; 42) without a bias toward flanker identities, whereas theories with an unfocused attentional spotlight that causes the substitution of flankers for the target (10, 11) would follow the failed predictions of our substitution model (Fig. 2). It is nonetheless conceivable that the attentional spotlight could perform a local averaging of crowded elements, and the observed averaging of feature positions could thus be attributed to either low-level or attention-based mechanisms.

Finally, the averaging processes observed in the present study appear remarkably similar to processes of contour integration (43), whereby spatially distinct elements are grouped into extended edges. In our stimuli in particular, averaging would have the effect of pulling the position of all feature elements in line with extended contours, consistent with a recent suggestion that crowding may underlie many reported phenomena in contour integration (44), including the insensitivity to curved contour structure in the periphery (45). The strong bias induced by collinear (Fig. 1C) compared with parallel features (Fig. S1) is also consistent with the tuning of contour integration for ‘snakes’ vs. ‘ladders’ (43). These interactions are likely to serve a beneficial role, with pooling along contours able to relieve the positional uncertainty inherent in the peripheral visual field (18). This finding raises the possibility that crowding processes may actively simplify image representation in the peripheral visual field by generating extended contours in an attempt to overcome both the low spatial sampling and high positional uncertainty within these regions.

## Materials and Methods

**Observers.** Two of the authors (J.A.G. and S.C.D.) and 1 naïve observer (D.K.) participated in each experiment. All had normal or corrected-to-normal visual acuity.

**Apparatus.** Experiments were programmed using MATLAB (MathWorks, Ltd.) running on a Macintosh computer with PsychToolbox software (46). Stimuli were presented on a CRT monitor (LaCie Electron Blue) (22), with a resolution of 1152 × 870 pixels and a 75-Hz refresh rate. The monitor was calibrated using a photometer and linearized in software, giving a mean and maximum luminance of 50 and 100 cd/m<sup>2</sup>, respectively. Stimuli were viewed with the dominant eye from 57 cm, with responses made using the keyboard (Experiment 1) or mouse (Experiment 2).

**Stimuli and Procedures.** Stimuli were white ‘cross-like’ elements (50% Weber contrast above mean luminance; Fig. 1A), with a stroke width equal to one-fifth the line length (equivalent to Sloan letters) and a total size set to twice the size-acuity thresholds for each observer. Targets were presented at 15° eccentricity

in the upper visual field, with a 0.7 × 0.7° cross presented at fixation. Under crowded conditions, 1 flanker was presented to the left and 1 to the right of targets. The centre-to-centre separation of the target and flankers was 2.5° in Experiment 1 [well within the tangential region of interference; (9, 23)], with additional target-flanker separations tested separately (Fig. S2). Test stimuli were presented for 200 ms, followed by a dense 20° × 5° masking array of cross stimuli with randomized feature positions for a further 200 ms.

Size-acuity thresholds were first estimated by having subjects judge the orientation (4AFC) of a cross-element configured to resemble a ‘T’ in 1 of 4 cardinal orientations. Stimulus size was determined using QUEST (47), which converged on 62.5% identification. This procedure was repeated 5 times for each observer, with the mean of these thresholds being 0.8° for both JAG and SCD, and 0.75° for DK. Stimuli were subsequently presented at twice these sizes.

Next, baseline thresholds for position discrimination were measured using a single cross. The x-position of the vertical line was fixed at the midpoint of the horizontal line (as in upright/inverted Ts), with the horizontal line of targets presented at 1 of 17 positions along the vertical line. Observers judged whether the presented position was above or below the stimulus midpoint (2AFC). Each position was repeated 10 times, with 5 blocks run according to the method of constant stimuli. Raw data were fit with cumulative Gaussian functions to determine the midpoint (50% upward) and threshold (75% upward) for each run. Thresholds for uncrowded position displacements were 0.14° for DK and SCD, and 0.18° for JAG. Midpoints did not differ significantly from 0, indicating a lack of bias in unflanked judgments of position.

To assess the effect of crowding on position encoding (Experiment 1), judgments of the vertical position of the horizontal line were made in the presence of 2 flanking elements. The horizontal lines of flankers were presented at 1 of 5 positions: the vertical midpoint of the stimulus (midflankers), and above or below the midpoint with either threshold or half-threshold displacements (up-flankers and down-flankers, respectively, using the uncrowded position thresholds obtained earlier). Unflanked thresholds were monitored throughout the experiment (allowing calculation of the frequency that subthreshold flanker displacements were likely to have been detected), resulting in 6 interleaved conditions. As before, the horizontal target line appeared at each position 10 times, with each block repeated 5 times. Data were again fit with cumulative Gaussian functions, with 95% confidence intervals determined for midpoints and thresholds using 1,000 repetitions of a bootstrap.

To determine the perceived position of target features (Experiment 2), horizontal target lines were again presented at a range of positions along the vertical line, either in isolation or crowded. Parameters in this experiment were equated across observers, with both positional displacements and stimulus size set to the maximum values of Experiment 1 (0.18° and 1.6°, respectively) and the target-flanker separation decreased to 1.75° to maximize crowding. Under crowded conditions, flanking horizontal lines were positioned either above or below the target position by a small (2× threshold, 0.36°) or large (4× threshold, 0.72°) displacement. To avoid positions exceeding stimulus dimensions, target positions varied between ± 0.4° of the stimulus midpoint in uncrowded and small flanker-offset conditions, or were focused in either the upper or lower half of the stimulus (again spanning 0.8°) with large flanker offsets, depending on the sign of offset. Following the mask, a response cross (identical to experimental stimuli) was presented 2° above fixation. Using the mouse, observers adjusted the horizontal line (initially randomly positioned) until it matched their percept of the target. Each run contained 3 presentations of each target position, with 21 positions per condition; 5 repetitions of each run were undertaken. Responses were smoothed with a Gaussian filter ( $\sigma = 1$  pixel/0.04°, the difference between each presented position). As responses to upward and downward flankers were mirror images of one another, the latter were inverted and both were pooled. Data were also pooled across observers, as all showed an identical pattern of performance.

**Model Simulations.** Predicted perceived positions (Experiment 2) were simulated using 2 versions of a 3-stage model, 1 employing a weighted average of target/flanker positions and the other a substitution of flanker features (both depicted in Fig. 2B). Each had 3 free parameters. The first and third stages were identical for each model. The first stage consisted of noisy estimates of flanker and target positions:

$$y = s + a\sigma_e \quad [1]$$

where  $s$  could represent  $s_t$  (the target-horizontal position), or  $s_{f1}$  and  $s_{f2}$  (flanker-horizontal positions),  $\sigma_e$  represents the Gaussian positional error and  $a$  is a free parameter—the magnitude of this error. Feature positions and Gaussian noise were clipped between ± 1, corresponding to the upper and lower extremes of the stimuli. When flankers were present, a second combi-

natorial stage was used, which could take 2 forms. In the weighted average model, combinations of the stimuli ( $y_c$ ) were:

$$y_c = \frac{(y_t w_t + y_{f1} w_{f1} + y_{f2} w_{f2})}{(w_t + w_{f1} + w_{f2})} \quad [2a]$$

where  $w_t$ ,  $w_{f1}$  and  $w_{f2}$  are the weights for the target and flankers, respectively. These weights summed to 1, with the proportion of the target weight as the second free parameter. The result is an average of the target and flanker positions that can be biased toward or away from the target representation. In contrast, substitution was produced using a conditional probability:

$$\text{if } \{p_{sub} y_f > (1 - p_{sub}) y_t\} \text{ then } \{y_c = y_f\} \text{ else } \{y_c = y_t\} \quad [2b]$$

Here,  $p_{sub}$  is the probability of flanker substitution, which is the second free parameter for the substitution model. Multiplication of this value with the strength of the target and flankers (i.e., their distance from the stimulus midpoint) determined the final report. The output of these 2 stages is shown in isolation in Fig. S4. Finally, the output of both models underwent reference repulsion: an inverse parabolic function pushed the estimated target position

away from the stimulus midpoint. This equation produced our final estimate of target position ( $y_{final}$ ):

$$y_{final} = y_c \pm r(1 - y_c)^2 + \sigma_d \quad [3]$$

Here,  $r$  is the magnitude of reference repulsion, the third free parameter. Additional decisional noise ( $\sigma_d$ ) was also introduced that was not parameterized, as it does not alter the output of the 2 models qualitatively, but was set to be proportional to the degree of repulsion.

Data obtained in Experiment 2 was used to set the 3 free parameters for each model ( $a$ ,  $r$ , and either  $w$  or  $p_{sub}$ ). The final parameter set minimized the least-squares error between model predictions (1,000 trials per point) and psychophysical data. Simulations of the 2AFC data in Experiment 1 were also produced (Fig. S3). Finally, simulations were run with a fourth parameter, which set the magnitude of an additional Gaussian error term in the second stage of both models. The inclusion of this term did not significantly improve model fits in either case, as determined by the Akaike Information Criterion (48). Because in both cases the 3-parameter models produced a smaller AIC than the 4-parameter fits, the inclusion of this second noise term was deemed unnecessary.

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# Supporting Information

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## SI Text

**Additional Data. Horizontal position judgments.** The primary result of Experiment 1 is that the crowding of positional information involves both an elevation in thresholds for positional discrimination and a systematic bias that shifts the perceived position of target elements into coalignment with flanking features. It is possible, however, that these effects are limited to collinear arrangements of stimulus features. For instance, the observed biases could be the result of observers incorrectly using the displaced flanker-horizontals as a reference for the up/down judgments, similar to a vernier task (1). Alternately, interactions between parallel and collinear elements may differ for more mechanistic reasons, particularly if crowding is related to contour integration processes (2), where the tuning for 'snake' vs. 'ladder' configurations differs markedly (3). We thus sought to assess the effect of crowding on position judgments along the horizontal axis of our stimuli.

Horizontal position judgments were assessed in a manner similar to that of Experiment 1, with observers required to judge whether the vertical line within target elements was positioned left or right of the stimulus midpoint. When flankers were present (Fig. S1A), the horizontal lines could either be centered or displaced to the left or right by a displacement equivalent to twice the threshold for uncrowded horizontal position judgments (assessed before the main experiment, as in Experiment 1). All other parameters were identical to those used previously. Example psychometric functions for the 4 conditions are displayed in Fig. S1B. As in Experiment 1, the presence of flankers led to psychometric functions that were shallower than those obtained with a single element, indicating that observers needed larger shifts to report the horizontal position of the target-vertical reliably. The presence of right-flankers also shifted the entire curve leftward (indicating more 'rightward' responses), which results in a negative midpoint value as the subjective midpoint (where observers were equally likely to say the target-horizontal was 'left' or 'right') is now located at a negative (leftward) position on the cross. The opposite pattern occurred with left-flankers.

Midpoint values are presented in Fig. S1C for each observer. In the absence of flankers, there was a tendency for observers to indicate 'right' responses more often than 'left', resulting in a slight (nonsignificant) negative value for uncrowded bias when averaged across observers. When 2 flankers were present on either side of the target, each with vertical lines positioned at the midpoint (midflankers), the bias was similarly near to zero (i.e., centered target-verticals still appeared centered when crowded). In contrast, flankers with positional displacements (equivalent to twice threshold) to the left of the stimulus midpoint produced an increase in 'leftward' responses, and a corresponding positive shift in bias. The same is true of the right-flankers condition, where negative bias is evident. As in Experiment 1, this result demonstrates that judgments were made in a direction consistent with flanking structure. Note also that the magnitude of this bias is close to that of Experiment 1, despite the positional displacements being double the maximum displacement tested previously.

Impairments in positional discrimination are also evident under these circumstances, with all flanker configurations again producing an equivalent level of threshold elevation (Fig. S1D). In contrast with Experiment 1, where thresholds were almost doubled, threshold elevation for these judgments was on average 8 times that of uncrowded thresholds. This marked performance

impairment occurred despite baseline thresholds being similar to, or lower than, the baseline thresholds of Experiment 1 ( $0.14^\circ$  for DK and SCD,  $0.12^\circ$  for JAG). There are 2 likely explanations for this difference. First, as the position judgments of Experiment 1 were made along the vertical dimension, the addition of flankers did nothing to alter the axis being judged. This arrangement differs for the horizontal position judgments, where the flankers were placed at the ends of the stimulus dimension being assessed. Their presence at these endpoints could thus add uncertainty regarding the horizontal dimension and cause performance impairments over those typically induced by crowding. That is, it is not simply the stimulus appearance that is being affected by the flankers, but also the reference points used to determine performance on the task. A second possibility is that interactions between parallel elements differ from those of collinear elements. This interpretation would be likely were crowding effects related to contour integration processes, given the strong contour integration seen for collinear 'snake' configurations and the poor integration operations for parallel 'ladder' elements (3).

Despite these differences, our data show that the effect of crowding on horizontal position judgments is qualitatively similar to that for vertical position judgments, with both an elevation in thresholds for position discrimination and a systematic bias that shifts the position of target elements to be more similar to those of the flanking elements.

**Effects of target-flanker separation.** Because of the difficulty in distinguishing crowding from effects such as masking (4, 5), surround suppression (6) or stochastic recalibration (7), it is important to establish that both random (threshold) and systematic (bias) effects observed in Experiment 1 arise from a common source. To this end, consider that 1 of the hallmarks of crowding is that the spatial extent of target-flanker interactions is approximately equal to half the target eccentricity regardless of target size (4, 8, 9), whereas masking and suppression scale with target size, regardless of eccentricity (10, 11). If both systematic and random effects arise from crowding, both should decrease in magnitude at the same rate as the target-flanker separation increases.

We examined this proposition using the same procedure as Experiment 1, measuring midpoint and threshold values for vertical position judgments (of the target-horizontal feature) at a range of target-flanker separations ( $1.75^\circ$  and between  $2.5^\circ$  and  $12.5^\circ$  in steps of either  $1.25^\circ$  for JAG or  $2.5^\circ$  for DK and SCD). Responses were pooled across up-flanker and down-flanker conditions, following the inversion of the latter. To provide a fair comparison of threshold and midpoint values, each data set was normalized by subtracting the appropriate baseline (uncrowded) value and dividing by the maximum. Cumulative Gaussian functions were fit to the resulting data, with the size of spatial interference zones determined as the midpoint of these functions, and 95% confidence intervals determined using a bootstrapping procedure (12).

In Fig. S2, midpoint and threshold values are plotted as a function of target-flanker separation. For each observer, the region of spatial interference is clearly similar for both values, with the half-width of these functions occurring between  $4\text{--}6^\circ$  ( $0.3\text{--}0.4$  times the target eccentricity of  $15^\circ$ ). These values are consistent with prior estimates of the spatial extent of crowding, particularly given the smaller region of interference along the tangential axis to fixation (8, 13). Importantly, this finding

suggests that both effects are likely to arise from a common mechanism associated with crowding.

**Additional Model Details.** The results of our experiments were simulated with two 3-stage models of crowding, one employing an average of target and flanker feature positions, and the other performing a substitution of flanker feature positions into the target. Fig. S4 shows the outputs of intermediate stages of these models to the stimuli used in Experiment 2. This figure complements the final outputs displayed in Fig. 2 in-text.

In the first stage (Fig. S4A), noisy estimates of the flanker and target feature positions were generated through the addition of Gaussian noise to the veridical positions (ranging between  $\pm 1$ ). When flankers were present, a second combinatorial stage was initiated in 1 of 2 forms. In the weighted average model (Fig. S4B), target and flanker feature positions were averaged with weights that summed to 1 (full equations are displayed in-text). At each presented position, this stage produced a unimodal distribution of 'perceived' positions that fell at intermediate values between the target and flanker positions. The final weighting scheme (determined using a least squares fit with the data) gave a weighting of 0.67 to the 2 flankers and 0.33 to the target, making the distributions peak at positions that were closer to the flankers. In contrast, substitution (Fig. S4C) was produced using a conditional probability whereby either the flanker or target positions were used on any given trial. For each presented position, this stage produced bimodal distributions with peaks at each of the target and flanker positions. Higher peaks occurred at the flanker positions because of the final value

for the  $p_{sub}$  variable (which determined the probability of flanker substitution) being biased toward the flankers (with a final value of 0.88). Because of the initial-stage positional uncertainty, these distributions sum to become unimodal with small flanker offsets (Left) and only become clearly bimodal with larger flanker offsets (Right). Finally, both models converged on a third stage of reference repulsion whereby an inverse parabolic function was used to push the final estimate of target position away from the stimulus midpoint. These final outputs are displayed in Fig. 2 in-text.

Simulations of the 2AFC data in Experiment 1 were also produced (Fig. S3) by performing the above steps and taking all  $y_{final}$  values  $>0$  as an 'upward' response and below zero as 'downward.' This simulation produced psychometric functions that largely matched those produced by our observers (Left), with midpoint values that also followed the structure of the flankers (Center). That is, flankers with both subthreshold- and threshold-level displacements below the stimulus midpoint produced an increase in 'downward' responses, and a corresponding positive shift in bias. The same is true for the up-flankers conditions, where negative bias is clearly evident for both displacements. The variability in flanker positions also produced the threshold elevation evident in our data (Right), with all configurations producing a broadly equivalent level of impairment. Both models can successfully simulate the responses of our observers in this experiment, and it is only when we assess the exact perceived position within cross elements that the predictions of the substitution model can be shown to be untenable (as in Experiment 2).

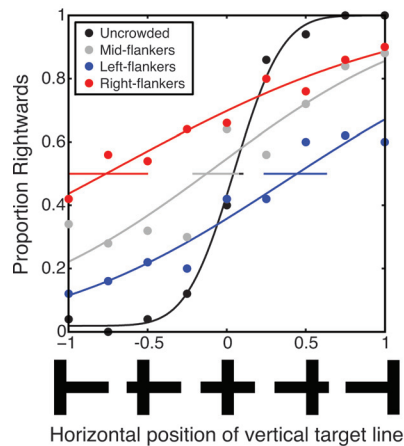
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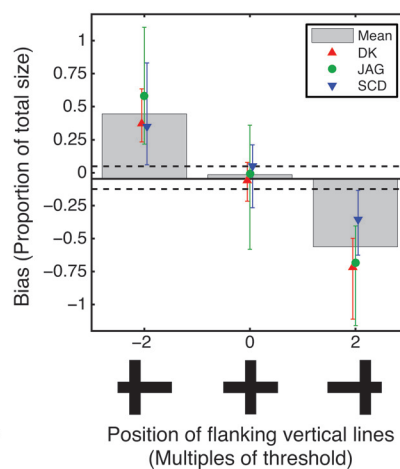
### A. Example stimuli (two sets)



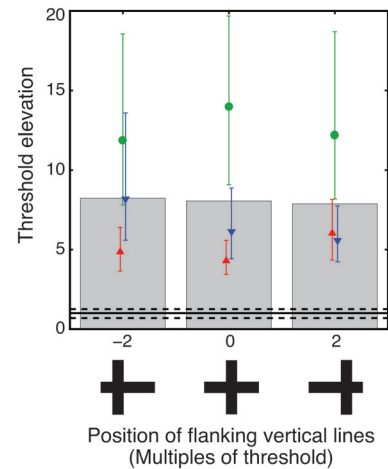
### B. Example data (DK)



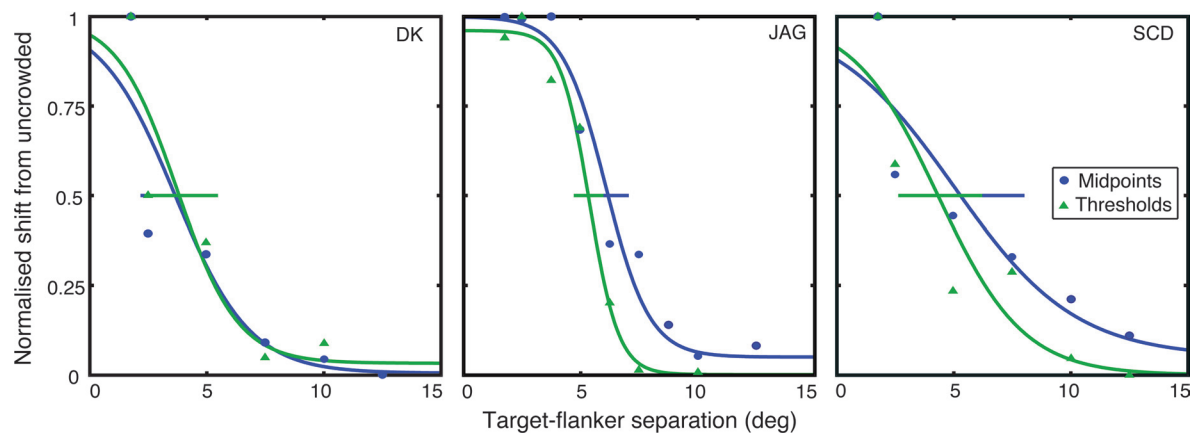
### C. Midpoint shifts (bias)



### D. Threshold elevation

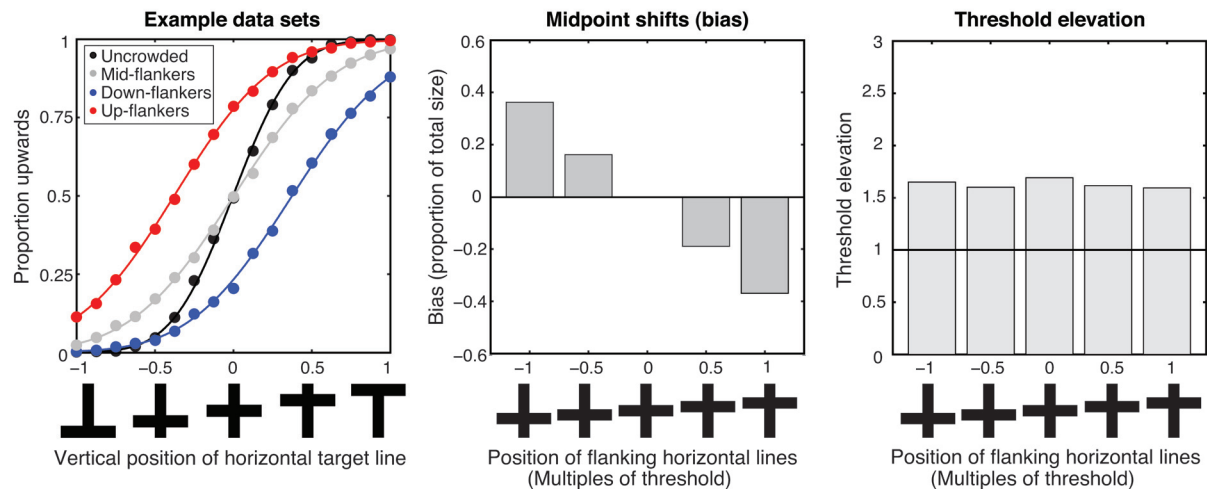


**Fig. S1.** The effects of crowding on encoding the horizontal position of vertical features. (A) Example stimuli, with the middle target-cross depicted with the vertical line centered along the horizontal axis. Flankers on the left have horizontal lines positioned to the left of the midpoint (left-flankers), with the converse right-flankers on the right. The effect of crowding can be seen by fixating near the bottom of the figure caption: target-verticals should appear shifted toward the flanker-verticals. (B) Data from 1 observer: the proportion of 'rightward' responses are plotted as a function of the horizontal position of the target-vertical (shown schematically on the abscissa), and fit with psychometric functions. Ninety-five percent confidence intervals are depicted around the midpoints, obtained from bootstrapping. Data are presented for targets in isolation (uncrowded), and crowded conditions with midflankers and left- and right-flankers offset by twice-threshold displacements. As in the main experiment, crowding both increases the slope and shifts psychometric functions to a degree that depends on flanking structure. (C) Midpoints of the psychometric functions (bias) for 3 observers (see legend) for each condition, expressed as a proportion of stimulus size (scaled to  $\pm 1$ ) and plotted as a function of the horizontal position of flanker-verticals (expressed in multiples of the uncrowded threshold for position discrimination). Mean data are presented as gray bars, with individual points superimposed; error bars (and hatched lines for uncrowded data) depict 95% confidence intervals. Positive shifts in flanker feature positions produce negative shifts in bias, indicating an increase in 'rightward' responses with right-flankers, and vice versa. (D) As in 1C for threshold elevation for the same 3 observers (see 1C legend). The presence of flankers produced a strong elevation of positional thresholds, regardless of their structure.

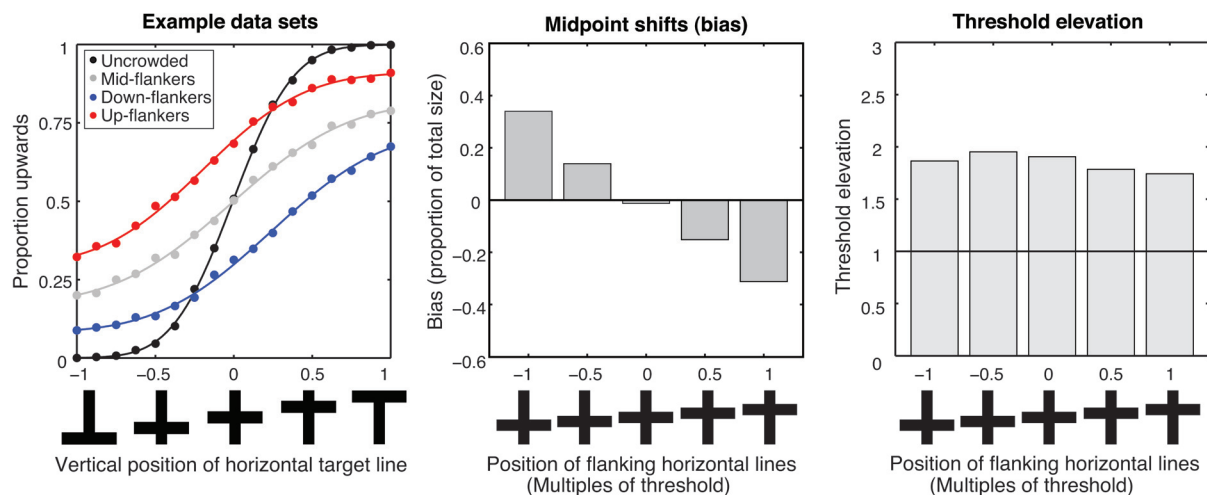


**Fig. S2.** Crowding-related changes in midpoint and threshold values as a function of target-flanker separation, plotted separately for each observer. Both datasets were normalized for comparison, yielding scores that varied between 0 (equivalent to uncrowded performance) and 1 (the maximum crowding effect observed). Cumulative Gaussian functions were fit to each dataset, with 95% confidence intervals for the half-width of spatial interference determined using a bootstrap procedure. Both factors produce equivalent spatial interference zones, with the half-width of functions occurring at a target-flanker separation between 4–6 ° for each observer.

## A. Averaging model



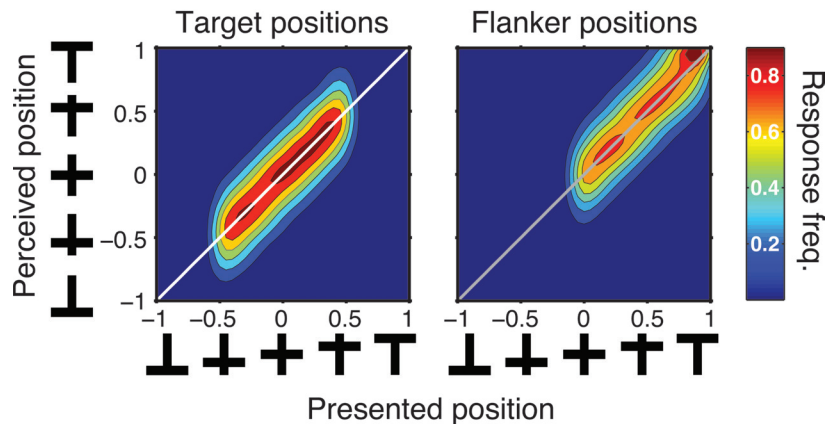
## B. Substitution model



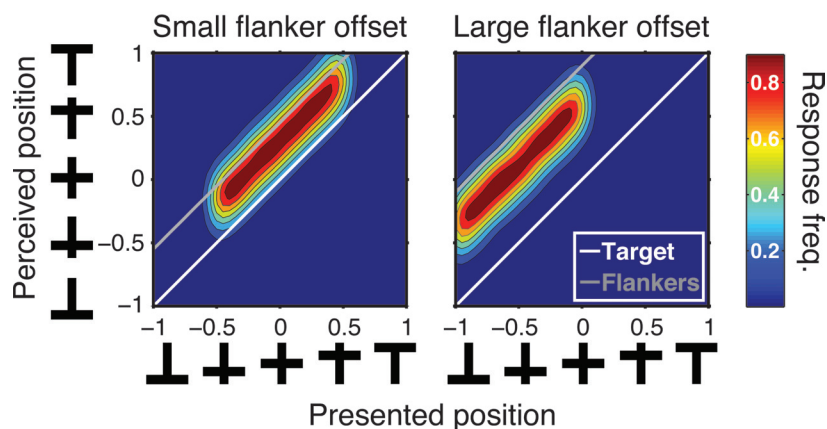
**Fig. S3.** Simulations of the responses in Experiment 1, with presentation conventions identical to those of Fig. 1 in-text. Parameters for both models were kept identical to those used in simulating the data of Experiment 2 (as in Fig. 2 in-text), save for a reduction in the magnitude of either flanker weighting or substitution (the second-stage parameters) because of the larger target-flanker separation in Experiment 1. (A) Simulations run with the averaging model. The *Left* depicts model responses to 4 of the 6 conditions (as in Fig. 1B), with each data point representing 1,000 trials. Results in the *Center* reflect the shift in bias for each condition (with positive values reflecting an increase in ‘downward’ responses, and vice versa), which can be seen to follow the same trend as in Fig. 1C. The threshold elevation produced under crowding (rightward panel) also follows a similar trend to that of Fig. 1D. (B) Simulations run with the substitution model. Compared with averaging, note that substitution produces a different effect on the extremes of the psychometric functions (*Left*), because there is always a base probability of substitution, functions will never reach their endpoints. This pattern contrasts with averaging, where the combination of up-flankers and up-targets will always allow 1 end of the function to reach either ceiling or floor in the extreme. Nonetheless, similar values of midpoint and threshold can be obtained under crowding, demonstrating that both models can satisfactorily account for the data of Experiment 1. It is only when we examine the distribution of perceived positions (Experiment 2) that the 2 models can be differentiated. Additionally, neither model adds noise at the second (combinatorial) stage, demonstrating that the combination of noisy positional estimates can produce threshold elevation in the absence of any additional noise from the crowding process.



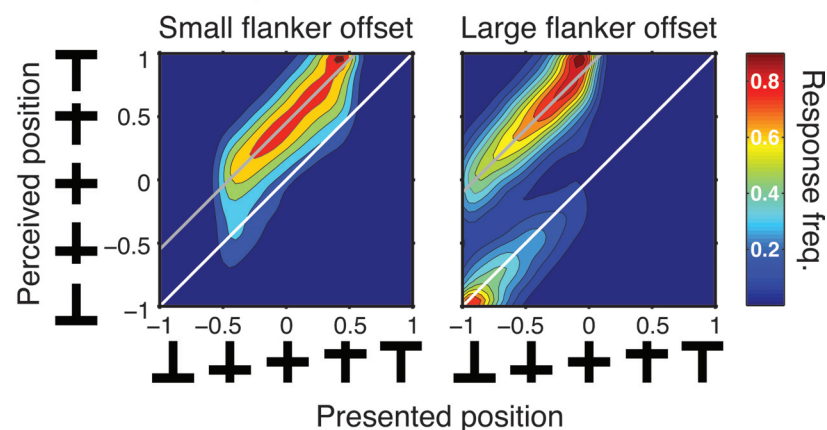
### A. First-stage output (initial position estimates)



### B. Second-stage output: Averaging model



### C. Second-stage output: Substitution model



**Fig. S4.** Output from intermediate stages of the 2 crowding models. (A) Output from the first stage, where the initial feature positions are computed for the target (left) and flankers (right, shown for the small flanker offsets condition). Both sets of features are represented in a close to veridical fashion, save for the corruption by early positional uncertainty. Note that the flanker positions are simply shifted versions of the target positions. Presentation conventions are identical to those of Fig. 2 in-text. (B) Output from the second (combinatorial) stage of the *averaging* model. In the absence of midpoint repulsion (computed in the third stage), these responses lie along an intermediate line running parallel to the target and flanker positions, with the precise position depending on the weighting values. (C) Output from the second stage of the *substitution* model. With a small flanker offset, positional noise causes the bimodal distributions to become unimodal, with a peak that lies close to the flanker positions. The true bimodality of these distributions can be seen clearly with large flanker offsets, where responses cluster around the target and flanker positions, before the effects of midpoint repulsion.